Posterior Analytics

By Aristotle

Based on the translation by G. R. G. Mure, with minor emendations by Daniel Kolak.

BOOK I

Chapter I

All instruction given or received by way of argument proceeds from pre-existent knowledge. This becomes evident upon a survey of all the species of such instruction. The mathematical sciences and all other speculative disciplines are acquired in this way, and so are the two forms of dialectical reasoning, syllogistic and inductive; for each of these latter make use of old knowledge to impart new, the syllogism assuming an audience that accepts its premises, induction exhibiting the universal as implicit in the clearly known particular. Again, the persuasion exerted by rhetorical arguments is in principle the same, since they use either example, a kind of induction, or enthymeme, a form of syllogism.

The pre-existent knowledge required is of two kinds. In some cases admission of the fact must be assumed, in others comprehension of the meaning of the term used, and sometimes both assumptions are essential. Thus, we assume that every predicate can be either truly affirmed or truly denied of any subject, and that 'triangle' means so and so; as regards 'unit' we have to make the double assumption of the meaning of the word and the existence of the thing. The reason is that these several objects are not equally obvious to us. Recognition of a truth may in some cases contain as factors both previous knowledge and also knowledge acquired simultaneously with that recognition-knowledge, this latter, of the particulars actually falling under the universal and therein already virtually known. For example, the student knew beforehand that the angles of every triangle are equal to two right angles; but it was only at the actual moment at which he was being led on to recognize this as true in the instance before him that he came to know 'this figure inscribed in the semicircle' to be a triangle. For some things (viz. the singulars finally reached which are not predicable of anything else as subject) are only learnt in this way, i.e. there is here no recognition through a middle of a minor term as subject to a major. Before he was led on to recognition or before he actually drew a conclusion, we should perhaps say that in a manner he knew, in a manner not.

If he did not in an unqualified sense of the term know the existence of this triangle, how could he know without qualification that its angles were equal to two right angles? No: clearly he knows not without qualification but only in the sense that he knows universally. If this distinction is not drawn, we are faced with the dilemma in the Meno: either a man will learn nothing or what he already knows; for we cannot accept the solution which some people offer. A man is asked, 'Do you, or do you not, know that every pair is even?' He says he does know it. The questioner then produces a particular pair, of the existence, and so a fortiori of the evenness, of which he was unaware. The solution which some people offer is to assert that they do not know that every pair is even, but only that everything which they know to be a pair is even: yet what they know to be even is that of which they have demonstrated evenness, i.e. what they made the subject of their premise, viz. not merely every triangle or number which they know to be such, but any and every number or triangle without reservation. For no premise is ever couched in the form 'every number which you know to be such', or 'every rectilinear figure which you know to be such': the predicate is always construed as applicable to any and every instance of the thing. On the other hand, I imagine there is nothing to prevent a man in one sense knowing what he is learning, in another not knowing it. The strange thing would be, not if in some sense he knew what he was learning, but if he were to know it in that precise sense and manner in which he was learning it.

Chapter 2

We suppose ourselves to possess unqualified scientific knowledge of a thing, as opposed to knowing it in the accidental way in which the sophist knows, when we think that we know the cause on which the fact depends, as the cause of that fact and of no other, and, further, that the fact could not be other than it is. Now that scientific knowing is something of this sort is evident-witness both those who falsely claim it and those who actually possess it, since the former merely imagine themselves to be, while the latter are also actually, in the condition described. Consequently the proper object of unqualified scientific knowledge is something which cannot be other than it is.

There may be another manner of knowing as well-that will be discussed later. What I now assert is that at all events we do know by demonstration. By demonstration I mean a syllogism productive of scientific knowledge, a syllogism, that is, the grasp of which is eo ipso such knowledge. Assuming then that my thesis as to the nature of scientific knowing is correct, the premises of demonstrated knowledge must be true, primary, immediate, better known than and prior to the conclusion, which is further related to them as effect to cause. Unless these conditions are satisfied, the basic truths will not be 'appropriate' to the conclusion. Syllogism there may indeed be without these conditions, but such syllogism, not being productive of scientific knowledge, will not be demonstration. The premises must be true: for that which is non-existent cannot be known-we cannot know, e.g. that the diagonal of a square is commensurate with its side. The premises must be primary and indemonstrable; otherwise they will require demonstration in order to be known, since to have knowledge, if it be not accidental knowledge, of things which are demonstrable, means precisely to have a demonstration of them. The premises must be the causes of the conclusion, better known than it, and prior to it; its causes, since we possess scientific knowledge of a thing only when we know its cause; prior, in order to be causes; antecedently known, this antecedent knowledge being not our mere understanding of the meaning, but knowledge of the fact as well. Now 'prior' and 'better known' are ambiguous terms, for there is a difference between what is prior and better known in the order of being and what is prior and better known to man. I mean that objects nearer to sense are prior and better known to man; objects without qualification prior and better known are those further from sense. Now the most universal causes are furthest from sense and particular causes are nearest to sense, and they are thus exactly opposed to one another. In saying that the premises of demonstrated knowledge must be primary, I mean that they must be the 'appropriate' basic truths, for I identify primary premise and basic truth. A 'basic truth' in a demonstration is an immediate proposition. An immediate proposition is one which has no other proposition prior to it. A proposition is either part of an enunciation, i.e. it predicates a single attribute of a single subject. If a proposition is dialectical, it assumes either part indifferently; if it is demonstrative, it lays down one part to the definite exclusion of the other because that part is true. The term 'enunciation' denotes either part of a contradiction indifferently. A contradiction is an opposition which of its own nature excludes a middle. The part of a contradiction which conjoins a predicate with a subject is an affirmation; the part disjoining them is a negation. I call an immediate basic truth of syllogism a 'thesis' when, though it is not susceptible of proof by the teacher, yet ignorance of it does not constitute a total bar to progress on the part of the pupil: one which the pupil must know if he is to learn anything whatever is an axiom. I call it an axiom because there are such truths and we give them the name of axioms par excellence. If a thesis assumes one part or the other of an enunciation, i.e. asserts either the existence or the non-existence of a subject, it is a hypothesis; if it does not so assert, it is a definition. Definition is a 'thesis' or a 'laying something down', since the arithmetician lays it down that to be a unit is to be quantitatively indivisible; but it is not a hypothesis, for to define what a unit is is not the same as to affirm its existence.

Now since the required ground of our knowledge-i.e. of our conviction-of a fact is the possession of such a syllogism as we call demonstration, and the ground of the syllogism is the facts constituting its premises, we must not only know the primary premises-some if not all of them-beforehand, but know them better than the conclusion: for the cause of an attribute's inherence in a subject always itself inheres in the subject more firmly than that attribute; e.g. the cause of our loving anything is dearer to us than the object of our love. So since the primary premises are the cause of our knowledge-i.e. of our conviction-it follows that we know them better-that is, are more convinced of them-than their consequences, precisely because of our knowledge of the latter is the effect of our knowledge of the premises. Now a man cannot believe in anything more than in the things he knows, unless he has either actual knowledge of it or something better than actual knowledge. But we are faced with this paradox if a student whose belief rests on demonstration has not prior knowledge; a man must believe in some, if not in all, of the basic truths more than in the conclusion. Moreover, if a man sets out to acquire the scientific knowledge that comes through demonstration, he must not only have a better knowledge of the basic truths and a firmer conviction of them than of the connection which is being demonstrated: more than this, nothing must be more certain or better known to him than these basic truths in their character as contradicting the fundamental premises which lead to the opposed and erroneous conclusion. For indeed the conviction of pure science must be unshakable.

Chapter 3

Some hold that, owing to the necessity of knowing the primary premises, there is no scientific knowledge. Others think there is, but that all truths are demonstrable. Neither doctrine is either true or a necessary deduction from the premises. The first school, assuming that there is no way of knowing other than by demonstration, maintain that an infinite regress is involved, on the ground that if behind the prior stands no primary, we could not know the posterior through the prior (wherein they are right, for one cannot traverse an infinite series): if on the other hand-they say-the series terminates and there are primary premises, yet these are unknowable because incapable of demonstration, which according to them is the only form of knowledge. And since thus one cannot know the primary premises, knowledge of the conclusions which follow from them is not pure scientific knowledge nor properly knowing at all, but rests on the mere supposition that the premises are true. The other party agree with them as regards knowing, holding that it is only possible by demonstration, but they see no difficulty in holding that all truths are demonstrated, on the ground that demonstration may be circular and reciprocal.

Our own doctrine is that not all knowledge is demonstrative: on the contrary, knowledge of the immediate premises is independent of demonstration. (The necessity of this is obvious; for since we must know the prior premises from which the demonstration is drawn, and since the regress must end in immediate truths, those truths must be indemonstrable.) Such, then, is our doctrine, and in addition we maintain that besides scientific knowledge there is its originative source which enables us to recognize the definitions.

Now demonstration must be based on premises prior to and better known than the conclusion; and the same things cannot simultaneously be both prior and posterior to one another: so circular demonstration is clearly not possible in the unqualified sense of 'demonstration', but only possible if 'demonstration' be extended to include that other method of argument which rests on a distinction between truths prior to us and truths without qualification prior, i.e. the method by which induction produces knowledge. But if we accept this extension of its meaning, our definition of unqualified knowledge will prove faulty; for there seem to be two kinds of it. Perhaps, however, the second form of demonstration, that which proceeds from truths better known to us, is not demonstration in the unqualified sense of the term.

The advocates of circular demonstration are not only faced with the difficulty we have just stated: in addition their theory reduces to the mere statement that if a thing exists, then it does exist-an easy way of proving anything. That this is so can be clearly shown by taking three terms, for to constitute the circle it makes no difference whether many terms or few or even only two are taken. Thus by direct proof, if A is, B must be; if B is, C must be; therefore if A is, C must be. Since then-by the circular proof-if A is, B must be, and if B is, A must be, A may be substituted for C above. Then 'if B is, A must be' = 'if B is, C must be', which above gave the conclusion 'if A is, C must be': but C and A have been identified. Consequently the upholders of circular demonstration are in the position of saying that if A is, A must be-a simple way of proving anything. Moreover, even such circular demonstration is impossible except in the case of attributes that imply one another, viz. 'peculiar' properties.

Now, it has been shown that the positing of one thing-be it one term or one premise-never involves a necessary consequent: two premises constitute the first and smallest foundation for drawing a conclusion at all and therefore a fortiori for the demonstrative syllogism of science. If, then, A is implied in B and C, and B and C are reciprocally implied in one another and in A, it is possible, as has been shown in my writings on the syllogism, to prove all the assumptions on which the original conclusion rested, by circular demonstration in the first figure. But it has also been shown that in the other figures either no conclusion is possible, or at least none which proves both the original premises. Propositions the terms of which are not convertible cannot be circularly demonstrated at all, and since convertible terms occur rarely in actual demonstrations, it is clearly frivolous and impossible to say that demonstration is reciprocal and that therefore everything can be demonstrated.

Chapter 4

Since the object of pure scientific knowledge cannot be other than it is, the truth obtained by demonstrative knowledge will be necessary. And since demonstrative knowledge is only present when we have a demonstration, it follows that demonstration is an inference from necessary premises. So we must consider what are the premises of demonstration-i.e. what is their character: and as a preliminary, let us define what we mean by an attribute 'true in every instance of its subject', an 'essential' attribute, and a 'commensurate and universal' attribute. I call 'true in every instance' what is truly predicable of all instances-not of one to the exclusion of others-and at all times, not at this or that time only; e.g. if animal is truly predicable of every instance of man, then if it be true to say 'this is a man', 'this is an animal' is also true, and if the one be true now the other is true now. A corresponding account holds if point is in every instance predicable as contained in line. There is evidence for this in the fact that the objection we raise against a proposition put to us as true in every instance is either an instance in which, or an occasion on which, it is not true. Essential attributes are (1) such as belong to their subject as elements in its essential nature (e.g. line thus belongs to triangle, point to line; for the very being or 'substance' of triangle and line is composed of these elements, which are contained in the formulae defining triangle and line): (2) such that, while they belong to certain subjects, the subjects to which they belong are contained in the attribute's own defining formula. Thus straight and curved belong to line, odd and even, prime and compound, square and oblong, to number; and also the formula defining any one of these attributes contains its subject-e.g. line or number as the case may be.

Extending this classification to all other attributes, I distinguish those that answer the above description as belonging essentially to their respective subjects; whereas attributes related in neither of these two ways to their subjects I call accidents or 'coincidents'; e.g. musical or white is a 'coincident' of animal.

Further (a) that is essential which is not predicated of a subject other than itself: e.g. 'the walking [thing]' walks and is white in virtue of being something else besides; whereas substance, in the sense of whatever signifies a 'this somewhat', is not what it is in virtue of being something else besides. Things, then, not predicated of a subject I call essential; things predicated of a subject I call accidental or 'coincidental'.

In another sense again (b) a thing consequentially connected with anything is essential; one not so connected is 'coincidental'. An example of the latter is 'While he was walking it lightened': the lightning was not due to his walking; it was, we should say, a coincidence. If, on the other hand, there is a consequential connection, the predication is essential; e.g. if a beast dies when its throat is being cut, then its death is also essentially connected with the cutting, because the cutting was the cause of death, not death a 'coincident' of the cutting.

So far then as concerns the sphere of connections scientifically known in the unqualified sense of that term, all attributes which (within that sphere) are essential either in the sense that their subjects are contained in them, or in the sense that they are contained in their subjects, are necessary as well as consequentially connected with their subjects. For it is impossible for them not to inhere in their subjects either simply or in the qualified sense that one or other of a pair of opposites must inhere in the subject; e.g. in line must be either straightness or curvature, in number either oddness or evenness. For within a single identical genus the contrary of a given attribute is either its privative or its contradictory; e.g. within number what is not odd is even, inasmuch as within this sphere even is a necessary consequent of not-odd. So, since any given predicate must be either affirmed or denied of any subject, essential attributes must inhere in their subjects of necessity.

Thus, then, we have established the distinction between the attribute which is 'true in every instance' and the 'essential' attribute.

I term 'commensurately universal' an attribute which belongs to every instance of its subject, and to every instance essentially and as such; from which it clearly follows that all commensurate universals inhere necessarily in their subjects. The essential attribute, and the attribute that belongs to its subject as such, are identical. E.g. point and straight belong to line essentially, for they belong to line as such; and triangle as such has two right angles, for it is essentially equal to two right angles.

An attribute belongs commensurately and universally to a subject when it can be shown to belong to any random instance of that subject and when the subject is the first thing to which it can be shown to belong. Thus, e.g. (1) the equality of its angles to two right angles is not a commensurately universal attribute of figure. For though it is possible to show that a figure has its angles equal to two right angles, this attribute cannot be demonstrated of any figure selected at haphazard, nor in demonstrating does one take a figure at random-a square is a figure but its angles are not equal to two right angles. On the other hand, any isosceles triangle has its angles equal to two right angles, yet isosceles triangle is not the primary subject of this attribute but triangle is prior. So whatever can be shown to have its angles equal to two right angles, or to possess any other attribute, in any random instance of itself and primarily-that is the first subject to which the predicate in question belongs commensurately and universally, and the demonstration, in the essential sense, of any predicate is the proof of it as belonging to this first subject commensurately and universally: while the proof of it as belonging to the other subjects to which it attaches is demonstration only in a secondary and unessential sense. Nor again (2) is equality to two right angles a commensurately universal attribute of isosceles; it is of wider application.

Chapter 5

We must not fail to observe that we often fall into error because our conclusion is not in fact primary and commensurately universal in the sense in which we think we prove it so. We make this mistake (1) when the subject is an individual or individuals above which there is no universal to be found: (2) when the subjects belong to different species and there is a higher universal, but it has no name: (3) when the subject which the demonstrator takes as a whole is really only a part of a larger whole; for then the demonstration will be true of the individual instances within the part and will hold in every instance of it, yet the demonstration will not be true of this subject primarily and commensurately and universally. When a demonstration is true of a subject primarily and commensurately and universally, that is to be taken to mean that it is true of a given subject primarily and as such. Case (3) may be thus exemplified. If a proof were given that perpendiculars to the same line are parallel, it might be supposed that lines thus perpendicular were the proper subject of the demonstration because being parallel is true of every instance of them. But it is not so, for the parallelism depends not on these angles being equal to one another because each is a right angle, but simply on their being equal to one another. An example of (1) would be as follows: if isosceles were the only triangle, it would be thought to have its angles equal to two right angles qua isosceles. An instance of (2) would be the law that proportionals alternate. Alternation used to be demonstrated separately of numbers, lines, solids, and durations, though it could have been proved of them all by a single demonstration. Because there was no single name to denote that in which numbers, lengths, durations, and solids are identical, and because they differed specifically from one another, this property was proved of each of them separately. To-day, however, the proof is commensurately universal, for they do not possess this attribute qua lines or qua numbers, but qua manifesting this generic character which they are postulated as possessing universally. Hence, even if one prove of each kind of triangle that its angles are equal to two right angles, whether by means of the same or different proofs; still, as long as one treats separately equilateral, scalene, and isosceles, one does not yet know, except sophistically, that triangle has its angles equal to two right angles, nor does one yet know that triangle has this property commensurately and universally, even if there is no other species of triangle but these. For one does not know that triangle as such has this property, nor even that 'all' triangles have it-unless 'all' means 'each taken singly': if 'all' means 'as a whole class', then, though there be none in which one does not recognize this property, one does not know it of 'all triangles'.

When, then, does our knowledge fail of commensurate universality, and when it is unqualified knowledge? If triangle be identical in essence with equilateral, i.e. with each or all equilaterals, then clearly we have unqualified knowledge: if on the other hand it be not, and the attribute belongs to equilateral qua triangle; then our knowledge fails of commensurate universality. 'But', it will be asked, 'does this attribute belong to the subject of which it has been demonstrated qua triangle or qua isosceles? What is the point at which the subject. to which it belongs is primary? (i.e. to what subject can it be demonstrated as belonging commensurately and universally?)' Clearly this point is the first term in which it is found to inhere as the elimination of inferior differentiae proceeds. Thus the angles of a brazen isosceles triangle are equal to two right angles: but eliminate brazen and isosceles and the attribute remains. 'But'-you may say-'eliminate figure or limit, and the attribute vanishes.' True, but figure and limit are not the first differentiae whose elimination destroys the attribute. 'Then what is the first?' If it is triangle, it will be in virtue of triangle that the attribute belongs to all the other subjects of which it is predicable, and triangle is the subject to which it can be demonstrated as belonging commensurately and universally.

Chapter 6

Demonstrative knowledge must rest on necessary basic truths; for the object of scientific knowledge cannot be other than it is. Now attributes attaching essentially to their subjects attach necessarily to them: for essential attributes are either elements in the essential nature of their subjects, or contain their subjects as elements in their own essential nature. (The pairs of opposites which the latter class includes are necessary because one member or the other necessarily inheres.) It follows from this that premises of the demonstrative syllogism must be connections essential in the sense explained: for all attributes must inhere essentially or else be accidental, and accidental attributes are not necessary to their subjects.

We must either state the case thus, or else premise that the conclusion of demonstration is necessary and that a demonstrated conclusion cannot be other than it is, and then infer that the conclusion must be developed from necessary premises. For though you may reason from true premises without demonstrating, yet if your premises are necessary you will assuredly demonstrate-in such necessity you have at once a distinctive character of demonstration. That demonstration proceeds from necessary premises is also indicated by the fact that the objection we raise against a professed demonstration is that a premise of it is not a necessary truth-whether we think it altogether devoid of necessity, or at any rate so far as our opponent's previous argument goes. This shows how naive it is to suppose one's basic truths rightly chosen if one starts with a proposition which is (1) popularly accepted and (2) true, such as the sophists' assumption that to know is the same as to possess knowledge. For (1) popular acceptance or rejection is no criterion of a basic truth, which can only be the primary law of the genus constituting the subject matter of the demonstration; and (2) not all truth is 'appropriate'.

A further proof that the conclusion must be the development of necessary premises is as follows. Where demonstration is possible, one who can give no account which includes the cause has no scientific knowledge. If, then, we suppose a syllogism in which, though A necessarily inheres in C, yet B, the middle term of the demonstration, is not necessarily connected with A and C, then the man who argues thus has no reasoned knowledge of the conclusion, since this conclusion does not owe its necessity to the middle term; for though the conclusion is necessary, the mediating link is a contingent fact. Or again, if a man is without knowledge now, though he still retains the steps of the argument, though there is no change in himself or in the fact and no lapse of memory on his part; then neither had he knowledge previously. But the mediating link, not being necessary, may have perished in the interval; and if so, though there be no change in him nor in the fact, and though he will still retain the steps of the argument, yet he has not knowledge, and therefore had not knowledge before. Even if the link has not actually perished but is liable to perish, this situation is possible and might occur. But such a condition cannot be knowledge.

When the conclusion is necessary, the middle through which it was proved may yet quite easily be non-necessary. You can in fact infer the necessary even from a non-necessary premise, just as you can infer the true from the not true. On the other hand, when the middle is necessary the conclusion must be necessary; just as true premises always give a true conclusion. Thus, if A is necessarily predicated of B and B of C, then A is necessarily predicated of C. But when the conclusion is nonnecessary the middle cannot be necessary either. Thus: let A be predicated non-necessarily of C but necessarily of B, and let B be a necessary predicate of C; then A too will be a necessary predicate of C, which by hypothesis it is not.

To sum up, then: demonstrative knowledge must be knowledge of a necessary nexus, and therefore must clearly be obtained through a necessary middle term; otherwise its possessor will know neither the cause nor the fact that his conclusion is a necessary connection. Either he will mistake the non-necessary for the necessary and believe the necessity of the conclusion without knowing it, or else he will not even believe it-in which case he will be equally ignorant, whether he actually infers the mere fact through middle terms or the reasoned fact and from immediate premises.

Of accidents that are not essential according to our definition of essential there is no demonstrative knowledge; for since an accident, in the sense in which I here speak of it, may also not inhere, it is impossible to prove its inherence as a necessary conclusion. A difficulty, however, might be raised as to why in dialectic, if the conclusion is not a necessary connection, such and such determinate premises should be proposed in order to deal with such and such determinate problems. Would not the result be the same if one asked any questions whatever and then merely stated one's conclusion? The solution is that determinate questions have to be put, not because the replies to them affirm facts which necessitate facts affirmed by the conclusion, but because these answers are propositions which if the answerer affirm, he must affirm the conclusion and affirm it with truth if they are true.

Since it is just those attributes within every genus which are essential and possessed by their respective subjects as such that are necessary it is clear that both the conclusions and the premises of demonstrations which produce scientific knowledge are essential. For accidents are not necessary: and, further, since accidents are not necessary one does not necessarily have reasoned knowledge of a conclusion drawn from them (this is so even if the accidental premises are invariable but not essential, as in proofs through signs; for though the conclusion be actually essential, one will not know it as essential nor know its reason); but to have reasoned knowledge of a conclusion is to know it through its cause. We may conclude that the middle must be consequentially connected with the minor, and the major with the middle.

Chapter 7

It follows that we cannot in demonstrating pass from one genus to another. We cannot, for instance, prove geometrical truths by arithmetic. For there are three elements in demonstration: (1) what is proved, the conclusion-an attribute inhering essentially in a genus; (2) the axioms, i.e. axioms which are premises of demonstration; (3) the subject-genus whose attributes, i.e. essential properties, are revealed by the demonstration. The axioms which are premises of demonstration may be identical in two or more sciences: but in the case of two different genera such as arithmetic and geometry you cannot apply arithmetical demonstration to the properties of magnitudes unless the magnitudes in question are numbers. How in certain cases transference is possible I will explain later.

Arithmetical demonstration and the other sciences likewise possess, each of them, their own genera; so that if the demonstration is to pass from one sphere to another, the genus must be either absolutely or to some extent the same. If this is not so, transference is clearly impossible, because the extreme and the middle terms must be drawn from the same genus: otherwise, as predicated, they will not be essential and will thus be accidents. That is why it cannot be proved by geometry that opposites fall under one science, nor even that the product of two cubes is a cube. Nor can the theorem of any one science be demonstrated by means of another science, unless these theorems are related as subordinate to superior (e.g. as optical theorems to geometry or harmonic theorems to arithmetic). Geometry again cannot prove of lines any property which they do not possess qua lines, i.e. in virtue of the fundamental truths of their peculiar genus: it cannot show, for example, that the straight line is the most beautiful of lines or the contrary of the circle; for these qualities do not belong to lines in virtue of their peculiar genus, but through some property which it shares with other genera.

Chapter 8

It is also clear that if the premises from which the syllogism proceeds are commensurately universal, the conclusion of such i.e. in the unqualified sense-must also be eternal. Therefore no attribute can be demonstrated nor known by strictly scientific knowledge to inhere in perishable things. The proof can only be accidental, because the attribute's connection with its perishable subject is not commensurately universal but temporary and special. If such a demonstration is made, one premise must be perishable and not commensurately universal (perishable because only if it is perishable will the conclusion be perishable; not commensurately universal, because the predicate will be predicable of some instances of the subject and not of others); so that the conclusion can only be that a fact is true at the moment-not commensurately and universally. The same is true of definitions, since a definition is either a primary premise or a conclusion of a demonstration, or else only differs from a demonstration in the order of its terms. Demonstration and science of merely frequent occurrences-e.g. of eclipse as happening to the moon-are, as such, clearly eternal: whereas so far as they are not eternal they are not fully commensurate. Other subjects too have properties attaching to them in the same way as eclipse attaches to the moon.

Chapter 9

It is clear that if the conclusion is to show an attribute inhering as such, nothing can be demonstrated except from its 'appropriate' basic truths. Consequently a proof even from true, indemonstrable, and immediate premises does not constitute knowledge. Such proofs are like Bryson's method of squaring the circle; for they operate by taking as their middle a common character-a character, therefore, which the subject may share with another-and consequently they apply equally to subjects different in kind. They therefore afford knowledge of an attribute only as inhering accidentally, not as belonging to its subject as such: otherwise they would not have been applicable to another genus.

Our knowledge of any attribute's connection with a subject is accidental unless we know that connection through the middle term in virtue of which it inheres, and as an inference from basic premises essential and 'appropriate' to the subject-unless we know, e.g. the property of possessing angles equal to two right angles as belonging to that subject in which it inheres essentially, and as inferred from basic premises essential and 'appropriate' to that subject: so that if that middle term also belongs essentially to the minor, the middle must belong to the same kind as the major and minor terms. The only exceptions to this rule are such cases as theorems in harmonics which are demonstrable by arithmetic. Such theorems are proved by the same middle terms as arithmetical properties, but with a qualification-the fact falls under a separate science (for the subject genus is separate), but the reasoned fact concerns the superior science, to which the attributes essentially belong. Thus, even these apparent exceptions show that no attribute is strictly demonstrable except from its 'appropriate' basic truths, which, however, in the case of these sciences have the requisite identity of character.

It is no less evident that the peculiar basic truths of each inhering attribute are indemonstrable; for basic truths from which they might be deduced would be basic truths of all that is, and the science to which they belonged would possess universal sovereignty. This is so because he knows better whose knowledge is deduced from higher causes, for his knowledge is from prior premises when it derives from causes themselves uncaused: hence, if he knows better than others or best of all, his knowledge would be science in a higher or the highest degree. But, as things are, demonstration is not transferable to another genus, with such exceptions as we have mentioned of the application of geometrical demonstrations to theorems in mechanics or optics, or of arithmetical demonstrations to those of harmonics.

It is hard to be sure whether one knows or not; for it is hard to be sure whether one's knowledge is based on the basic truths appropriate to each attribute-the differentia of true knowledge. We think we have scientific knowledge if we have reasoned from true and primary premises. But that is not so: the conclusion must be homogeneous with the basic facts of the science.

Chapter 10

I call the basic truths of every genus those elements in it the existence of which cannot be proved. As regards both these primary truths and the attributes dependent on them the meaning of the name is assumed. The fact of their existence as regards the primary truths must be assumed; but it has to be proved of the remainder, the attributes. Thus we assume the meaning alike of unity, straight, and triangular; but while as regards unity and magnitude we assume also the fact of their existence, in the case of the remainder proof is required.

Of the basic truths used in the demonstrative sciences some are peculiar to each science, and some are common, but common only in the sense of analogous, being of use only in so far as they fall within the genus constituting the province of the science in question.

Peculiar truths are, e.g. the definitions of line and straight; common truths are such as 'take equals from equals and equals remain'. Only so much of these common truths is required as falls within the genus in question: for a truth of this kind will have the same force even if not used generally but applied by the geometer only to magnitudes, or by the arithmetician only to numbers. Also peculiar to a science are the subjects the existence as well as the meaning of which it assumes, and the essential attributes of which it investigates, e.g. in arithmetic units, in geometry points and lines. Both the existence and the meaning of the subjects are assumed by these sciences; but of their essential attributes only the meaning is assumed. For example arithmetic assumes the meaning of odd and even, square and cube, geometry that of incommensurable, or of deflection or verging of lines, whereas the existence of these attributes is demonstrated by means of the axioms and from previous conclusions as premises. Astronomy too proceeds in the same way. For indeed every demonstrative science has three elements: (1) that which it posits, the subject genus whose essential attributes it examines; (2) the so-called axioms, which are primary premises of its demonstration; (3) the attributes, the meaning of which it assumes. Yet some sciences may very well pass over some of these elements; e.g. we might not expressly posit the existence of the genus if its existence were obvious (for instance, the existence of hot and cold is more evident than that of number); or we might omit to assume expressly the meaning of the attributes if it were well understood. In the way the meaning of axioms, such as 'Take equals from equals and equals remain', is well known and so not expressly assumed. Nevertheless in the nature of the case the essential elements of demonstration are three: the subject, the attributes, and the basic premises.

That which expresses necessary self-grounded fact, and which we must necessarily believe, is distinct both from the hypotheses of a science and from illegitimate postulate-I say 'must believe', because all syllogism, and therefore a fortiori demonstration, is addressed not to the spoken word, but to the discourse within the soul, and though we can always raise objections to the spoken word, to the inward discourse we cannot always object. That which is capable of proof but assumed by the teacher without proof is, if the pupil believes and accepts it, hypothesis, though only in a limited sense hypothesis-that is, relatively to the pupil; if the pupil has no opinion or a contrary opinion on the matter, the same assumption is an illegitimate postulate. Therein lies the distinction between hypothesis and illegitimate postulate: the latter is the contrary of the pupil's opinion, demonstrable, but assumed and used without demonstration.

The definition-viz. those which are not expressed as statements that anything is or is not-are not hypotheses: but it is in the premises of a science that its hypotheses are contained. Definitions require only to be understood, and this is not hypothesis-unless it be contended that the pupil's hearing is also an hypothesis required by the teacher. Hypotheses, on the contrary, postulate facts on the being of which depends the being of the fact inferred. Nor are the geometer's hypotheses false, as some have held, urging that one must not employ falsehood and that the geometer is uttering falsehood in stating that the line which he draws is a foot long or straight, when it is actually neither. The truth is that the geometer does not draw any conclusion from the being of the particular line of which he speaks, but from what his diagrams symbolize. A further distinction is that all hypotheses and illegitimate postulates are either universal or particular, whereas a definition is neither.

Chapter 11

So demonstration does not necessarily imply the being of Forms nor a One beside a Many, but it does necessarily imply the possibility of truly predicating one of many; since without this possibility we cannot save the universal, and if the universal goes, the middle term goes with it, and so demonstration becomes impossible. We conclude, then, that there must be a single identical term unequivocally predicable of a number of individuals.

The law that it is impossible to affirm and deny simultaneously the same predicate of the same subject is not expressly posited by any demonstration except when the conclusion also has to be expressed in that form; in which case the proof lays down as its major premise that the major is truly affirmed of the middle but falsely denied. It makes no difference, however, if we add to the middle, or again to the minor term, the corresponding negative. For grant a minor term of which it is true to predicate man-even if it be also true to predicate not-man of it--still grant simply that man is animal and not not-animal, and the conclusion follows: for it will still be true to say that Callias--even if it be also true to say that not-Callias--is animal and not not-animal. The reason is that the major term is predicable not only of the middle, but of something other than the middle as well, being of wider application; so that the conclusion is not affected even if the middle is extended to cover the original middle term and also what is not the original middle term.

The law that every predicate can be either truly affirmed or truly denied of every subject is posited by such demonstration as uses reductio ad impossibile, and then not always universally, but so far as it is requisite; within the limits, that is, of the genus-the genus, I mean (as I have already explained), to which the man of science applies his demonstrations. In virtue of the common elements of demonstration-I mean the common axioms which are used as premises of demonstration, not the subjects nor the attributes demonstrated as belonging to them-all the sciences have communion with one another, and in communion with them all is dialectic and any science which might attempt a universal proof of axioms such as the law of excluded middle, the law that the subtraction of equals from equals leaves equal remainders, or other axioms of the same kind. Dialectic has no definite sphere of this kind, not being confined to a single genus. Otherwise its method would not be interrogative; for the interrogative method is barred to the demonstrator, who cannot use the opposite facts to prove the same nexus. This was shown in my work on the syllogism.

Chapter 12

If a syllogistic question is equivalent to a proposition embodying one of the two sides of a contradiction, and if each science has its peculiar propositions from which its peculiar conclusion is developed, then there is such a thing as a distinctively scientific question, and it is the interrogative form of the premises from which the 'appropriate' conclusion of each science is developed. Hence it is clear that not every question will be relevant to geometry, nor to medicine, nor to any other science: only those questions will be geometrical which form premises for the proof of the theorems of geometry or of any other science, such as optics, which uses the same basic truths as geometry. Of the other sciences the like is true. Of these questions the geometer is bound to give his account, using the basic truths of geometry in conjunction with his previous conclusions; of the basic truths the geometer, as such, is not bound to give any account. The like is true of the other sciences. There is a limit, then, to the questions which we may put to each man of science; nor is each man of science bound to answer all inquiries on each several subject, but only such as fall within the defined field of his own science. If, then, in controversy with a geometer qua geometer the disputant confines himself to geometry and proves anything from geometrical premises, he is clearly to be applauded; if he goes outside these he will be at fault, and obviously cannot even refute the geometer except accidentally. One should therefore not discuss geometry among those who are not geometers, for in such a company an unsound argument will pass unnoticed. This is correspondingly true in the other sciences.

Since there are 'geometrical' questions, does it follow that there are also distinctively 'ungeometrical' questions? Further, in each special science-geometry for instance-what kind of error is it that may vitiate questions, and yet not exclude them from that science? Again, is the erroneous conclusion one constructed from premises opposite to the true premises, or is it formal fallacy though drawn from geometrical premises? Or, perhaps, the erroneous conclusion is due to the drawing of premises from another science; e.g. in a geometrical controversy a musical question is distinctively ungeometrical, whereas the notion that parallels meet is in one sense geometrical, being ungeometrical in a different fashion: the reason being that 'ungeometrical', like 'unrhythmical', is equivocal, meaning in the one case not geometry at all, in the other bad geometry? It is this error, i.e. error based on premises of this kind-'of' the science but false-that is the contrary of science. In mathematics the formal fallacy is not so common, because it is the middle term in which the ambiguity lies, since the major is predicated of the whole of the middle and the middle of the whole of the minor (the predicate of course never has the prefix 'all'); and in mathematics one can, so to speak, see these middle terms with an intellectual vision, while in dialectic the ambiguity may escape detection. E.g. 'Is every circle a figure?' A diagram shows that this is so, but the minor premise 'Are epics circles?' is shown by the diagram to be false.

If a proof has an inductive minor premise, one should not bring an 'objection' against it. For since every premise must be applicable to a number of cases (otherwise it will not be true in every instance, which, since the syllogism proceeds from universals, it must be), then assuredly the same is true of an 'objection'; since premises and 'objections' are so far the same that anything which can be validly advanced as an 'objection' must be such that it could take the form of a premise, either demonstrative or dialectical. On the other hand, arguments formally illogical do sometimes occur through taking as middles mere attributes of the major and minor terms. An instance of this is Caeneus' proof that fire increases in geometrical proportion: 'Fire', he argues, 'increases rapidly, and so does geometrical proportion'. There is no syllogism so, but there is a syllogism if the most rapidly increasing proportion is geometrical and the most rapidly increasing proportion is attributable to fire in its motion. Sometimes, no doubt, it is impossible to reason from premises predicating mere attributes: but sometimes it is possible, though the possibility is overlooked. If false premises could never give true conclusions 'resolution' would be easy, for premises and conclusion would in that case inevitably reciprocate. I might then argue thus: let A be an existing fact; let the existence of A imply such and such facts actually known to me to exist, which we may call B. I can now, since they reciprocate, infer A from B.

Reciprocation of premises and conclusion is more frequent in mathematics, because mathematics takes definitions, but never an accident, for its premises-a second characteristic distinguishing mathematical reasoning from dialectical disputations.

A science expands not by the interposition of fresh middle terms, but by the apposition of fresh extreme terms. E.g. A is predicated of B, B of C, C of D, and so indefinitely. Or the expansion may be lateral: e.g. one major A, may be proved of two minors, C and E. Thus let A represent number-a number or number taken indeterminately; B determinate odd number; C any particular odd number. We can then predicate A of C. Next let D represent determinate even number, and E even number. Then A is predicable of E.

Chapter 13

Knowledge of the fact differs from knowledge of the reasoned fact. To begin with, they differ within the same science and in two ways: (1) when the premises of the syllogism are not immediate (for then the proximate cause is not contained in them-a necessary condition of knowledge of the reasoned fact): (2) when the premises are immediate, but instead of the cause the better known of the two reciprocals is taken as the middle; for of two reciprocally predicable terms the one which is not the cause may quite easily be the better known and so become the middle term of the demonstration. Thus (2) (a) you might prove as follows that the planets are near because they do not twinkle: let C be the planets, B not twinkling, A proximity. Then B is predicable of C; for the planets do not twinkle. But A is also predicable of B, since that which does not twinkle is near--we must take this truth as having been reached by induction or sense-perception. Therefore A is a necessary predicate of C; so that we have demonstrated that the planets are near. This syllogism, then, proves not the reasoned fact but only the fact; since they are not near because they do not twinkle, but, because they are near, do not twinkle. The major and middle of the proof, however, may be reversed, and then the demonstration will be of the reasoned fact. Thus: let C be the planets, B proximity, A not twinkling. Then B is an attribute of C, and A-not twinkling-of B. Consequently A is predicable of C, and the syllogism proves the reasoned fact, since its middle term is the proximate cause. Another example is the inference that the moon is spherical from its manner of waxing. Thus: since that which so waxes is spherical, and since the moon so waxes, clearly the moon is spherical. Put in this form, the syllogism turns out to be proof of the fact, but if the middle and major be reversed it is proof of the reasoned fact; since the moon is not spherical because it waxes in a certain manner, but waxes in such a manner because it is spherical. (Let C be the moon, B spherical, and A waxing.) Again (b), in cases where the cause and the effect are not reciprocal and the effect is the better known, the fact is demonstrated but not the reasoned fact. This also occurs (1) when the middle falls outside the major and minor, for here too the strict cause is not given, and so the demonstration is of the fact, not of the reasoned fact. For example, the question 'Why does not a wall breathe?' might be answered, 'Because it is not an animal'; but that answer would not give the strict cause, because if not being an animal causes the absence of respiration, then being an animal should be the cause of respiration, according to the rule that if the negation of causes the non-inherence of y, the affirmation of x causes the inherence of y; e.g. if the disproportion of the hot and cold elements is the cause of ill health, their proportion is the cause of health; and conversely, if the assertion of x causes the inherence of y, the negation of x must cause y's non-inherence. But in the case given this consequence does not result; for not every animal breathes. A syllogism with this kind of cause takes place in the second figure. Thus: let A be animal, B respiration, C wall. Then A is predicable of all B (for all that breathes is animal), but of no C; and consequently B is predicable of no C; that is, the wall does not breathe. Such causes are like far-fetched explanations, which precisely consist in making the cause too remote, as in Anacharsis' account of why the Scythians have no flute-players; namely because they have no vines.

Thus, then, do the syllogism of the fact and the syllogism of the reasoned fact differ within one science and according to the position of the middle terms. But there is another way too in which the fact and the reasoned fact differ, and that is when they are investigated respectively by different sciences. This occurs in the case of problems related to one another as subordinate and superior, as when optical problems are subordinated to geometry, mechanical problems to stereometry, harmonic problems to arithmetic, the data of observation to astronomy. (Some of these sciences bear almost the same name; e.g. mathematical and nautical astronomy, mathematical and acoustical harmonics.) Here it is the business of the empirical observers to know the fact, of the mathematicians to know the reasoned fact; for the latter are in possession of the demonstrations giving the causes, and are often ignorant of the fact: just as we have often a clear insight into a universal, but through lack of observation are ignorant of some of its particular instances. These connections have a perceptible existence though they are manifestations of forms. For the mathematical sciences concern forms: they do not demonstrate properties of a substratum, since, even though the geometrical subjects are predicable as properties of a perceptible substratum, it is not as thus predicable that the mathematician demonstrates properties of them. As optics is related to geometry, so another science is related to optics, namely the theory of the rainbow. Here knowledge of the fact is within the province of the natural philosopher, knowledge of the reasoned fact within that of the optician, either qua optician or qua mathematical optician. Many sciences not standing in this mutual relation enter into it at points; e.g. medicine and geometry: it is the physician's business to know that circular wounds heal more slowly, the geometer's to know the reason why.

Chapter 14

Of all the figures the most scientific is the first. Thus, it is the vehicle of the demonstrations of all the mathematical sciences, such as arithmetic, geometry, and optics, and practically all of all sciences that investigate causes: for the syllogism of the reasoned fact is either exclusively or generally speaking and in most cases in this figure-a second proof that this figure is the most scientific; for grasp of a reasoned conclusion is the primary condition of knowledge. Thirdly, the first is the only figure which enables us to pursue knowledge of the essence of a thing. In the second figure no affirmative conclusion is possible, and knowledge of a thing's essence must be affirmative; while in the third figure the conclusion can be affirmative, but cannot be universal, and essence must have a universal character: e.g. man is not two-footed animal in any qualified sense, but universally. Finally, the first figure has no need of the others, while it is by means of the first that the other two figures are developed, and have their intervals close-packed until immediate premises are reached.

Clearly, therefore, the first figure is the primary condition of knowledge.

Chapter 15

Just as an attribute A may (as we saw) be atomically connected with a subject B, so its disconnection may be atomic. I call 'atomic' connections or disconnections which involve no intermediate term; since in that case the connection or disconnection will not be mediated by something other than the terms themselves. It follows that if either A or B, or both A and B, have a genus, their disconnection cannot be primary. Thus: let C be the genus of A. Then, if C is not the genus of B-for A may well have a genus which is not the genus of B-there will be a syllogism proving A's disconnection from B thus:

all A is C, no B is C, therefore no B is A. Or if it is B which has a genus D, we have

all B is D, no D is A, therefore no B is A, by syllogism; and the proof will be similar if both A and B have a genus. That the genus of A need not be the genus of B and vice versa, is shown by the existence of mutually exclusive coordinate series of predication. If no term in the series ACD...is predicable of any term in the series BEF...,and if G-a term in the former series-is the genus of A, clearly G will not be the genus of B; since, if it were, the series would not be mutually exclusive. So also if B has a genus, it will not be the genus of A. If, on the other hand, neither A nor B has a genus and A does not inhere in B, this disconnection must be atomic. If there be a middle term, one or other of them is bound to have a genus, for the syllogism will be either in the first or the second figure. If it is in the first, B will have a genus-for the premise containing it must be affirmative: if in the second, either A or B indifferently, since syllogism is possible if either is contained in a negative premise, but not if both premises are negative.

Hence it is clear that one thing may be atomically disconnected from another, and we have stated when and how this is possible.

Chapter 16

Ignorance-defined not as the negation of knowledge but as a positive state of mind-is error produced by inference.

(1) Let us first consider propositions asserting a predicate's immediate connection with or disconnection from a subject. Here, it is true, positive error may befall one in alternative ways; for it may arise where one directly believes a connection or disconnection as well as where one's belief is acquired by inference. The error, however, that consists in a direct belief is without complication; but the error resulting from inference-which here concerns us-takes many forms. Thus, let A be atomically disconnected from all B: then the conclusion inferred through a middle term C, that all B is A, will be a case of error produced by syllogism. Now, two cases are possible. Either (a) both premises, or (b) one premise only, may be false. (a) If neither A is an attribute of any C nor C of any B, whereas the contrary was posited in both cases, both premises will be false. (C may quite well be so related to A and B that C is neither subordinate to A nor a universal attribute of B: for B, since A was said to be primarily disconnected from B, cannot have a genus, and A need not necessarily be a universal attribute of all things. Consequently both premises may be false.) On the other hand, (b) one of the premises may be true, though not either indifferently but only the major A-C since, B having no genus, the premise C-B will always be false, while A-C may be true. This is the case if, for example, A is related atomically to both C and B; because when the same term is related atomically to more terms than one, neither of those terms will belong to the other. It is, of course, equally the case if A-C is not atomic.

Error of attribution, then, occurs through these causes and in this form only-for we found that no syllogism of universal attribution was possible in any figure but the first. On the other hand, an error of non-attribution may occur either in the first or in the second figure. Let us therefore first explain the various forms it takes in the first figure and the character of the premises in each case.

(c) It may occur when both premises are false; e.g. supposing A atomically connected with both C and B, if it be then assumed that no C is and all B is C, both premises are false.

(d) It is also possible when one is false. This may be either premise indifferently. A-C may be true, C-B false-A-C true because A is not an attribute of all things, C-B false because C, which never has the attribute A, cannot be an attribute of B; for if C-B were true, the premise A-C would no longer be true, and besides if both premises were true, the conclusion would be true. Or again, C-B may be true and A-C false; e.g. if both C and A contain B as genera, one of them must be subordinate to the other, so that if the premise takes the form No C is A, it will be false. This makes it clear that whether either or both premises are false, the conclusion will equally be false.

In the second figure the premises cannot both be wholly false; for if all B is A, no middle term can be with truth universally affirmed of one extreme and universally denied of the other: but premises in which the middle is affirmed of one extreme and denied of the other are the necessary condition if one is to get a valid inference at all. Therefore if, taken in this way, they are wholly false, their contraries conversely should be wholly true. But this is impossible. On the other hand, there is nothing to prevent both premises being partially false; e.g. if actually some A is C and some B is C, then if it is premised that all A is C and no B is C, both premises are false, yet partially, not wholly, false. The same is true if the major is made negative instead of the minor. Or one premise may be wholly false, and it may be either of them. Thus, supposing that actually an attribute of all A must also be an attribute of all B, then if C is yet taken to be a universal attribute of all but universally non-attributable to B, C-A will be true but C-B false. Again, actually that which is an attribute of no B will not be an attribute of all A either; for if it be an attribute of all A, it will also be an attribute of all B, which is contrary to supposition; but if C be nevertheless assumed to be a universal attribute of A, but an attribute of no B, then the premise C-B is true but the major is false. The case is similar if the major is made the negative premise. For in fact what is an attribute of no A will not be an attribute of any B either; and if it be yet assumed that C is universally non-attributable to A, but a universal attribute of B, the premise C-A is true but the minor wholly false. Again, in fact it is false to assume that that which is an attribute of all B is an attribute of no A, for if it be an attribute of all B, it must be an attribute of some A. If then C is nevertheless assumed to be an attribute of all B but of no A, C-B will be true but C-A false.

It is thus clear that in the case of atomic propositions erroneous inference will be possible not only when both premises are false but also when only one is false.

Chapter 17

In the case of attributes not atomically connected with or disconnected from their subjects, (a) (i) as long as the false conclusion is inferred through the 'appropriate' middle, only the major and not both premises can be false. By 'appropriate middle' I mean the middle term through which the contradictory-i.e. the true-conclusion is inferable. Thus, let A be attributable to B through a middle term C: then, since to produce a conclusion the premise C-B must be taken affirmatively, it is clear that this premise must always be true, for its quality is not changed. But the major A-C is false, for it is by a change in the quality of A-C that the conclusion becomes its contradictory-i.e. true. Similarly (ii) if the middle is taken from another series of predication; e.g. suppose D to be not only contained within A as a part within its whole but also predicable of all B. Then the premise D-B must remain unchanged, but the quality of A-D must be changed; so that D-B is always true, A-D always false. Such error is practically identical with that which is inferred through the 'appropriate' middle. On the other hand, (b) if the conclusion is not inferred through the 'appropriate' middle-(i) when the middle is subordinate to A but is predicable of no B, both premises must be false, because if there is to be a conclusion both must be posited as asserting the contrary of what is actually the fact, and so posited both become false: e.g. suppose that actually all D is A but no B is D; then if these premises are changed in quality, a conclusion will follow and both of the new premises will be false. When, however, (ii) the middle D is not subordinate to A, A-D will be true, D-B false-A-D true because A was not subordinate to D, D-B false because if it had been true, the conclusion too would have been true; but it is ex hypothesi false.

When the erroneous inference is in the second figure, both premises cannot be entirely false; since if B is subordinate to A, there can be no middle predicable of all of one extreme and of none of the other, as was stated before. One premise, however, may be false, and it may be either of them. Thus, if C is actually an attribute of both A and B, but is assumed to be an attribute of A only and not of B, C-A will be true, C-B false: or again if C be assumed to be attributable to B but to no A, C-B will be true, C-A false.

We have stated when and through what kinds of premises error will result in cases where the erroneous conclusion is negative. If the conclusion is affirmative, (a) (i) it may be inferred through the 'appropriate' middle term. In this case both premises cannot be false since, as we said before, C-B must remain unchanged if there is to be a conclusion, and consequently A-C, the quality of which is changed, will always be false. This is equally true if (ii) the middle is taken from another series of predication, as was stated to be the case also with regard to negative error; for D-B must remain unchanged, while the quality of A-D must be converted, and the type of error is the same as before.

(b) The middle may be inappropriate. Then (i) if D is subordinate to A, A-D will be true, but D-B false; since A may quite well be predicable of several terms no one of which can be subordinated to another. If, however, (ii) D is not subordinate to A, obviously A-D, since it is affirmed, will always be false, while D-B may be either true or false; for A may very well be an attribute of no D, whereas all B is D, e.g. no science is animal, all music is science. Equally well A may be an attribute of no D, and D of no B. It emerges, then, that if the middle term is not subordinate to the major, not only both premises but either singly may be false.

Thus we have made it clear how many varieties of erroneous inference are liable to happen and through what kinds of premises they occur, in the case both of immediate and of demonstrable truths.

Chapter 18

It is also clear that the loss of any one of the senses entails the loss of a corresponding portion of knowledge, and that, since we learn either by induction or by demonstration, this knowledge cannot be acquired. Thus demonstration develops from universals, induction from particulars; but since it is possible to familiarize the pupil with even the so-called mathematical abstractions only through induction-i.e. only because each subject genus possesses, in virtue of a determinate mathematical character, certain properties which can be treated as separate even though they do not exist in isolation-it is consequently impossible to come to grasp universals except through induction. But induction is impossible for those who have not sense-perception. For it is sense-perception alone which is adequate for grasping the particulars: they cannot be objects of scientific knowledge, because neither can universals give us knowledge of them without induction, nor can we get it through induction without sense-perception.

Chapter 19

Every syllogism is effected by means of three terms. One kind of syllogism serves to prove that A inheres in C by showing that A inheres in B and B in C; the other is negative and one of its premises asserts one term of another, while the other denies one term of another. It is clear, then, that these are the fundamentals and so-called hypotheses of syllogism. Assume them as they have been stated, and proof is bound to follow-proof that A inheres in C through B, and again that A inheres in B through some other middle term, and similarly that B inheres in C. If our reasoning aims at gaining credence and so is merely dialectical, it is obvious that we have only to see that our inference is based on premises as credible as possible: so that if a middle term between A and B is credible though not real, one can reason through it and complete a dialectical syllogism. If, however, one is aiming at truth, one must be guided by the real connections of subjects and attributes. Thus: since there are attributes which are predicated of a subject essentially or naturally and not coincidentally-not, that is, in the sense in which we say 'That white (thing) is a man', which is not the same mode of predication as when we say 'The man is white': the man is white not because he is something else but because he is man, but the white is man because 'being white' coincides with 'humanity' within one substratum-therefore there are terms such as are naturally subjects of predicates. Suppose, then, C such a term not itself attributable to anything else as to a subject, but the proximate subject of the attribute B--i.e. so that B-C is immediate; suppose further E related immediately to F, and F to B. The first question is, must this series terminate, or can it proceed to infinity? The second question is as follows: Suppose nothing is essentially predicated of A, but A is predicated primarily of H and of no intermediate prior term, and suppose H similarly related to G and G to B; then must this series also terminate, or can it too proceed to infinity? There is this much difference between the questions: the first is, is it possible to start from that which is not itself attributable to anything else but is the subject of attributes, and ascend to infinity? The second is the problem whether one can start from that which is a predicate but not itself a subject of predicates, and descend to infinity? A third question is, if the extreme terms are fixed, can there be an infinity of middles? I mean this: suppose for example that A inheres in C and B is intermediate between them, but between B and A there are other middles, and between these again fresh middles; can these proceed to infinity or can they not? This is the equivalent of inquiring, do demonstrations proceed to infinity, i.e. is everything demonstrable? Or do ultimate subject and primary attribute limit one another?

I hold that the same questions arise with regard to negative conclusions and premises: viz. if A is attributable to no B, then either this predication will be primary, or there will be an intermediate term prior to B to which a is not attributable-G, let us say, which is attributable to all B-and there may still be another term H prior to G, which is attributable to all G. The same questions arise, I say, because in these cases too either the series of prior terms to which a is not attributable is infinite or it terminates.

One cannot ask the same questions in the case of reciprocating terms, since when subject and predicate are convertible there is neither primary nor ultimate subject, seeing that all the reciprocals qua subjects stand in the same relation to one another, whether we say that the subject has an infinity of attributes or that both subjects and attributes-and we raised the question in both cases-are infinite in number. These questions then cannot be asked-unless, indeed, the terms can reciprocate by two different modes, by accidental predication in one relation and natural predication in the other.

Chapter 20

Now, it is clear that if the predications terminate in both the upward and the downward direction (by 'upward' I mean the ascent to the more universal, by 'downward' the descent to the more particular), the middle terms cannot be infinite in number. For suppose that A is predicated of F, and that the intermediates-call them BB'B"...-are infinite, then clearly you might descend from and find one term predicated of another ad infinitum, since you have an infinity of terms between you and F; and equally, if you ascend from F, there are infinite terms between you and A. It follows that if these processes are impossible there cannot be an infinity of intermediates between A and F. Nor is it of any effect to urge that some terms of the series AB...F are contiguous so as to exclude intermediates, while others cannot be taken into the argument at all: whichever terms of the series B...I take, the number of intermediates in the direction either of A or of F must be finite or infinite: where the infinite series starts, whether from the first term or from a later one, is of no moment, for the succeeding terms in any case are infinite in number.

Chapter 21

Further, if in affirmative demonstration the series terminates in both directions, clearly it will terminate too in negative demonstration. Let us assume that we cannot proceed to infinity either by ascending from the ultimate term (by 'ultimate term' I mean a term such as was, not itself attributable to a subject but itself the subject of attributes), or by descending towards an ultimate from the primary term (by 'primary term' I mean a term predicable of a subject but not itself a subject). If this assumption is justified, the series will also terminate in the case of negation. For a negative conclusion can be proved in all three figures. In the first figure it is proved thus: no B is A, all C is B. In packing the interval B-C we must reach immediate propositions--as is always the case with the minor premise--since B-C is affirmative. As regards the other premise it is plain that if the major term is denied of a term D prior to B, D will have to be predicable of all B, and if the major is denied of yet another term prior to D, this term must be predicable of all D. Consequently, since the ascending series is finite, the descent will also terminate and there will be a subject of which A is primarily non-predicable. In the second figure the syllogism is, all A is B, no C is B, no C is A. If proof of this is required, plainly it may be shown either in the first figure as above, in the second as here, or in the third. The first figure has been discussed, and we will proceed to display the second, proof by which will be as follows: all B is D, no C is D..., since it is required that B should be a subject of which a predicate is affirmed. Next, since D is to be proved not to belong to C, then D has a further predicate which is denied of C. Therefore, since the succession of predicates affirmed of an ever higher universal terminates, the succession of predicates denied terminates too.

The third figure shows it as follows: all B is A, some B is not C. Therefore some A is not C. This premise, i.e. C-B, will be proved either in the same figure or in one of the two figures discussed above. In the first and second figures the series terminates. If we use the third figure, we shall take as premises, all E is B, some E is not C, and this premise again will be proved by a similar prosyllogism. But since it is assumed that the series of descending subjects also terminates, plainly the series of more universal non-predicables will terminate also. Even supposing that the proof is not confined to one method, but employs them all and is now in the first figure, now in the second or third-even so the regress will terminate, for the methods are finite in number, and if finite things are combined in a finite number of ways, the result must be finite.

Thus it is plain that the regress of middles terminates in the case of negative demonstration, if it does so also in the case of affirmative demonstration. That in fact the regress terminates in both these cases may be made clear by the following dialectical considerations.

Chapter 22

In the case of predicates constituting the essential nature of a thing, it clearly terminates, seeing that if definition is possible, or in other words, if essential form is knowable, and an infinite series cannot be traversed, predicates constituting a thing's essential nature must be finite in number. But as regards predicates generally we have the following prefatory remarks to make. (1) We can affirm without falsehood 'the white (thing) is walking', and that big (thing) is a log'; or again, 'the log is big', and 'the man walks'. But the affirmation differs in the two cases. When I affirm 'the white is a log', I mean that something which happens to be white is a log-not that white is the substratum in which log inheres, for it was not qua white or qua a species of white that the white (thing) came to be a log, and the white (thing) is consequently not a log except incidentally. On the other hand, when I affirm 'the log is white', I do not mean that something else, which happens also to be a log, is white (as I should if I said 'the musician is white,' which would mean 'the man who happens also to be a musician is white'); on the contrary, log is here the substratum-the substratum which actually came to be white, and did so qua wood or qua a species of wood and qua nothing else.

If we must lay down a rule, let us entitle the latter kind of statement predication, and the former not predication at all, or not strict but accidental predication. 'White' and 'log' will thus serve as types respectively of predicate and subject.

We shall assume, then, that the predicate is invariably predicated strictly and not accidentally of the subject, for on such predication demonstrations depend for their force. It follows from this that when a single attribute is predicated of a single subject, the predicate must affirm of the subject either some element constituting its essential nature, or that it is in some way qualified, quantified, essentially related, active, passive, placed, or dated.

(2) Predicates which signify substance signify that the subject is identical with the predicate or with a species of the predicate. Predicates not signifying substance which are predicated of a

subject not identical with themselves or with a species of themselves are accidental or coincidental; e.g. white is a coincident of man, seeing that man is not identical with white or a species of white, but rather with animal, since man is identical with a species of animal. These predicates which do not signify substance must be predicates of some other subject, and nothing can be white which is not also other than white. The Forms we can dispense with, for they are mere sound without sense; and even if there are such things, they are not relevant to our discussion, since demonstrations are concerned with predicates such as we have defined.

(3) If A is a quality of B, B cannot be a quality of A-a quality of a quality. Therefore A and B cannot be predicated reciprocally of one another in strict predication: they can be affirmed without falsehood of one another, but not genuinely predicated of each other. For one alternative is that they should be substantially predicated of one another, i.e. B would become the genus or differentia of A-the predicate now become subject. But it has been shown that in these substantial predications neither the ascending predicates nor the descending subjects form an infinite series; e.g. neither the series, man is biped, biped is animal, &c., nor the series predicating animal of man, man of Callias, Callias of a further. subject as an element of its essential nature, is infinite. For all such substance is definable, and an infinite series cannot be traversed in thought: consequently neither the ascent nor the descent is infinite, since a substance whose predicates were infinite would not be definable. Hence they will not be predicated each as the genus of the other; for this would equate a genus with one of its own species. Nor (the other alternative) can a quale be reciprocally predicated of a quale, nor any term belonging to an adjectival category of another such term, except by accidental predication; for all such predicates are coincidents and are predicated of substances. On the other hand-in proof of the impossibility of an infinite ascending series-every predication displays the subject as somehow qualified or quantified or as characterized under one of the other adjectival categories, or else is an element in its substantial nature: these latter are limited in number, and the number of the widest kinds under which predications fall is also limited, for every predication must exhibit its subject as somehow qualified, quantified, essentially related, acting or suffering, or in some place or at some time.

I assume first that predication implies a single subject and a single attribute, and secondly that predicates which are not substantial are not predicated of one another. We assume this because such predicates are all coincidents, and though some are essential coincidents, others of a different type, yet we maintain that all of them alike are predicated of some substratum and that a coincident is never a substratum-since we do not class as a coincident anything which does not owe its designation to its being something other than itself, but always hold that any coincident is predicated of some substratum other than itself, and that another group of coincidents may have a different substratum. Subject to these assumptions then, neither the ascending nor the descending series of predication in which a single attribute is predicated of a single subject is infinite. For the subjects of which coincidents are predicated are as many as the constitutive elements of each individual substance, and these we have seen are not infinite in number, while in the ascending series are contained those constitutive elements with their coincidents-both of which are finite. We conclude that there is a given subject (D) of which some attribute (C) is primarily predicable; that there must be an attribute (B) primarily predicable of the first attribute, and that the series must end with a term (A) not predicable of any term prior to the last subject of which it was predicated (B), and of which no term prior to it is predicable.

The argument we have given is one of the so-called proofs; an alternative proof follows. Predicates so related to their subjects that there are other predicates prior to them predicable of those subjects are demonstrable; but of demonstrable propositions one cannot have something better than knowledge, nor can one know them without demonstration. Secondly, if a consequent is only known through an antecedent (viz. premises prior to it) and we neither know this antecedent nor have something better than knowledge of it, then we shall not have scientific knowledge of the consequent. Therefore, if it is possible through demonstration to know anything without qualification and not merely as dependent on the acceptance of certain premises-i.e. hypothetically-the series of intermediate predications must terminate. If it does not terminate, and beyond any predicate taken as higher than another there remains another still higher, then every predicate is demonstrable. Consequently, since these demonstrable predicates are infinite in number and therefore cannot be traversed, we shall not know them by demonstration. If, therefore, we have not something better than knowledge of them, we cannot through demonstration have unqualified but only hypothetical science of anything.

As dialectical proofs of our contention these may carry conviction, but an analytic process will show more briefly that neither the ascent nor the descent of predication can be infinite in the demonstrative sciences which are the object of our investigation. Demonstration proves the inherence of essential attributes in things. Now attributes may be essential for two reasons: either because they are elements in the essential nature of their subjects, or because their subjects are elements in their essential nature. An example of the latter is odd as an attribute of number-though it is number's attribute, yet number itself is an element in the definition of odd; of the former, multiplicity or the indivisible, which are elements in the definition of number. In neither kind of attribution can the terms be infinite. They are not infinite where each is related to the term below it as odd is to number, for this would mean the inherence in odd of another attribute of odd in whose nature odd was an essential element: but then number will be an ultimate subject of the whole infinite chain of attributes, and be an element in the definition of each of them. Hence, since an infinity of attributes such as contain their subject in their definition cannot inhere in a single thing, the ascending series is equally finite. Note, moreover, that all such attributes must so inhere in the ultimate subject-e.g. its attributes in number and number in them-as to be commensurate with the subject and not of wider extent. Attributes which are essential elements in the nature of their subjects are equally finite: otherwise definition would be impossible. Hence, if all the attributes predicated are essential and these cannot be infinite, the ascending series will terminate, and consequently the descending series too.

If this is so, it follows that the intermediates between any two terms are also always limited in number. An immediately obvious consequence of this is that demonstrations necessarily involve basic truths, and that the contention of some-referred to at the outset-that all truths are demonstrable is mistaken. For if there are basic truths, (a) not all truths are demonstrable, and (b) an infinite regress is impossible; since if either (a) or (b) were not a fact, it would mean that no interval was immediate and indivisible, but that all intervals were divisible. This is true because a conclusion is demonstrated by the interposition, not the apposition, of a fresh term. If such interposition could continue to infinity there might be an infinite number of terms between any two terms; but this is impossible if both the ascending and descending series of predication terminate; and of this fact, which before was shown dialectically, analytic proof has now been given.

Chapter 23

It is an evident corollary of these conclusions that if the same attribute A inheres in two terms C and D predicable either not at all, or not of all instances, of one another, it does not always belong to them in virtue of a common middle term. Isosceles and scalene possess the attribute of having their angles equal to two right angles in virtue of a common middle; for they possess it in so far as they are both a certain kind of figure, and not in so far as they differ from one another. But this is not always the case: for, were it so, if we take B as the common middle in virtue of which A inheres in C and D, clearly B would inhere in C and D through a second common middle, and this in turn would inhere in C and D through a third, so that between two terms an infinity of intermediates would fall-an impossibility. Thus it need not always be in virtue of a common middle term that a single attribute inheres in several subjects, since there must be immediate intervals. Yet if the attribute to be proved common to two subjects is to be one of their essential attributes, the middle terms involved must be within one subject genus and be derived from the same group of immediate premises; for we have seen that processes of proof cannot pass from one genus to another.

It is also clear that when A inheres in B, this can be demonstrated if there is a middle term. Further, the 'elements' of such a conclusion are the premises containing the middle in question, and they are identical in number with the middle terms, seeing that the immediate propositions-or at least such immediate propositions as are universal-are the 'elements'. If, on the other hand, there is no middle term, demonstration ceases to be possible: we are on the way to the basic truths. Similarly if A does not inhere in B, this can be demonstrated if there is a middle term or a term prior to B in which A does not inhere: otherwise there is no demonstration and a basic truth is reached. There are, moreover, as many 'elements' of the demonstrated conclusion as there are middle terms, since it is propositions containing these middle terms that are the basic premises on which the demonstration rests; and as there are some indemonstrable basic truths asserting that 'this is that' or that 'this inheres in that', so there are others denying that 'this is that' or that 'this inheres in that'-in fact some basic truths will affirm and some will deny being.

When we are to prove a conclusion, we must take a primary essential predicate-suppose it C-of the subject B, and then suppose A similarly predicable of C. If we proceed in this manner, no proposition or attribute which falls beyond A is admitted in the proof: the interval is constantly condensed until subject and predicate become indivisible, i.e. one. We have our unit when the premise becomes immediate, since the immediate premise alone is a single premise in the unqualified sense of 'single'. And as in other spheres the basic element is simple but not identical in all-in a system of weight it is the mina, in music the quarter-tone, and so on--so in syllogism the unit is an immediate premise, and in the knowledge that demonstration gives it is an intuition. In syllogisms, then, which prove the inherence of an attribute, nothing falls outside the major term. In the case of negative syllogisms on the other hand, (1) in the first figure nothing falls outside the major term whose inherence is in question; e.g. to prove through a middle C that A does not inhere in B the premises required are, all B is C, no C is A. Then if it has to be proved that no C is A, a middle must be found between and C; and this procedure will never vary.

(2) If we have to show that E is not D by means of the premises, all D is C; no E, or not all E, is C; then the middle will never fall beyond E, and E is the subject of which D is to be denied in the conclusion.

(3) In the third figure the middle will never fall beyond the limits of the subject and the attribute denied of it.

Chapter 24

Since demonstrations may be either commensurately universal or particular, and either affirmative or negative; the question arises, which form is the better? And the same question may be put in regard to so-called 'direct' demonstration and reductio ad impossibile. Let us first examine the commensurately universal and the particular forms, and when we have cleared up this problem proceed to discuss 'direct' demonstration and reductio ad impossibile.

The following considerations might lead some minds to prefer particular demonstration.

(1) The superior demonstration is the demonstration which gives us greater knowledge (for this is the ideal of demonstration), and we have greater knowledge of a particular individual when we know it in itself than when we know it through something else; e.g. we know Coriscus the musician better when we know that Coriscus is musical than when we know only that man is musical, and a like argument holds in all other cases. But commensurately universal demonstration, instead of proving that the subject itself actually is x, proves only that something else is x- e.g. in attempting to prove that isosceles is x, it proves not that isosceles but only that triangle is x- whereas particular demonstration proves that the subject itself is x. The demonstration, then, that a subject, as such, possesses an attribute is superior. If this is so, and if the particular rather than the commensurately universal forms demonstrates, particular demonstration is superior.

(2) The universal has not a separate being over against groups of singulars. Demonstration nevertheless creates the opinion that its function is conditioned by something like this-some separate entity belonging to the real world; that, for instance, of triangle or of figure or number, over against particular triangles, figures, and numbers. But demonstration which touches the real and will not mislead is superior to that which moves among unrealities and is delusory. Now commensurately universal demonstration is of the latter kind: if we engage in it we find ourselves reasoning after a fashion well illustrated by the argument that the proportionate is what answers to the definition of some entity which is neither line, number, solid, nor plane, but a proportionate apart from all these. Since, then, such a proof is characteristically commensurate and universal, and less touches reality than does particular demonstration, and creates a false opinion, it will follow that commensurate and universal is inferior to particular demonstration.

We may retort thus. (1) The first argument applies no more to commensurate and universal than to particular demonstration. If equality to two right angles is attributable to its subject not qua isosceles but qua triangle, he who knows that isosceles possesses that attribute knows the subject as qua itself possessing the attribute, to a less degree than he who knows that triangle has that attribute. To sum up the whole matter: if a subject is proved to possess qua triangle an attribute which it does not in fact possess qua triangle, that is not demonstration: but if it does possess it qua triangle the rule applies that the greater knowledge is his who knows the subject as possessing its attribute qua that in virtue of which it actually does possess it. Since, then, triangle is the wider term, and there is one identical definition of triangle-i.e. the term is not equivocal- and since equality to two right angles belongs to all triangles, it is isosceles qua triangle and not triangle qua isosceles which has its angles so related. It follows that he who knows a connection universally has greater knowledge of it as it in fact is than he who knows the particular; and the inference is that commensurate and universal is superior to particular demonstration.

(2) If there is a single identical definition i.e. if the commensurate universal is unequivocal-then the universal will possess being not less but more than some of the particulars, inasmuch as it is universals which comprise the imperishable, particulars that tend to perish.

(3) Because the universal has a single meaning, we are not therefore compelled to suppose that in these examples it has being as a substance apart from its particulars-any more than we need make a similar supposition in the other cases of unequivocal universal predication, viz. where the predicate signifies not substance but quality, essential relatedness, or action. If such a supposition is entertained, the blame rests not with the demonstration but with the hearer.

(4) Demonstration is syllogism that proves the cause, i.e. the reasoned fact, and it is rather the commensurate universal than the particular which is causative (as may be shown thus: that which possesses an attribute through its own essential nature is itself the cause of the inherence, and the commensurate universal is primary; hence the commensurate universal is the cause). Consequently commensurately universal demonstration is superior as more especially proving the cause, that is the reasoned fact.

(5) Our search for the reason ceases, and we think that we know, when the coming to be or existence of the fact before us is not due to the coming to be or existence of some other fact, for the last step of a search thus conducted is eo ipso the end and limit of the problem. Thus: 'Why did he come?' 'To get the money-wherewith to pay a debt-that he might thereby do what was right.' When in this regress we can no longer find an efficient or final cause, we regard the last step of it as the end of the coming-or being or coming to be-and we regard ourselves as then only having full knowledge of the reason why he came.

If, then, all causes and reasons are alike in this respect, and if this is the means to full knowledge in the case of final causes such as we have exemplified, it follows that in the case of the other causes also full knowledge is attained when an attribute no longer inheres because of something else. Thus, when we learn that exterior angles are equal to four right angles because they are the exterior angles of an isosceles, there still remains the question 'Why has isosceles this attribute?' and its answer 'Because it is a triangle, and a triangle has it because a triangle is a rectilinear figure.' If rectilinear figure possesses the property for no further reason, at this point we have full knowledge-but at this point our knowledge has become commensurately universal, and so we conclude that commensurately universal demonstration is superior.

(6) The more demonstration becomes particular the more it sinks into an indeterminate manifold, while universal demonstration tends to the simple and determinate. But objects so far as they are an indeterminate manifold are unintelligible, so far as they are determinate, intelligible: they are therefore intelligible rather in so far as they are universal than in so far as they are particular. From this it follows that universals are more demonstrable: but since relative and correlative increase concomitantly, of the more demonstrable there will be fuller demonstration. Hence the commensurate and universal form, being more truly demonstration, is the superior.

(7) Demonstration which teaches two things is preferable to demonstration which teaches only one. He who possesses commensurately universal demonstration knows the particular as well,

but he who possesses particular demonstration does not know the universal. So that this is an additional reason for preferring commensurately universal demonstration. And there is yet this further argument:

(8) Proof becomes more and more proof of the commensurate universal as its middle term approaches nearer to the basic truth, and nothing is so near as the immediate premise which is itself the basic truth. If, then, proof from the basic truth is more accurate than proof not so derived, demonstration which depends more closely on it is more accurate than demonstration which is less closely dependent. But commensurately universal demonstration is characterized by this closer dependence, and is therefore superior. Thus, if A had to be proved to inhere in D, and the middles were B and C, B being the higher term would render the demonstration which it mediated the more universal.

Some of these arguments, however, are dialectical. The clearest indication of the precedence of commensurately universal demonstration is as follows: if of two propositions, a prior and a posterior, we have a grasp of the prior, we have a kind of knowledge-a potential grasp-of the posterior as well. For example, if one knows that the angles of all triangles are equal to two right angles, one knows in a sense-potentially-that the isosceles' angles also are equal to two right angles, even if one does not know that the isosceles is a triangle; but to grasp this posterior proposition is by no means to know the commensurate universal either potentially or actually. Moreover, commensurately universal demonstration is through and through intelligible; particular demonstration issues in sense-perception.

Chapter 25

The preceding arguments constitute our defense of the superiority of commensurately universal to particular demonstration. That affirmative demonstration excels negative may be shown as follows.

(1) We may assume the superiority ceteris paribus of the demonstration which derives from fewer postulates or hypotheses-in short from fewer premises; for, given that all these are equally well known, where they are fewer knowledge will be more speedily acquired, and that is a desideratum. The argument implied in our contention that demonstration from fewer assumptions is superior may be set out in universal form as follows. Assuming that in both cases alike the middle terms are known, and that middles which are prior are better known than such as are posterior, we may suppose two demonstrations of the inherence of A in E, the one proving it through the middles B, C and D, the other through F and G. Then A-D is known to the same degree as A-E (in the second proof), but A-D is better known than and prior to A-E (in the first proof); since A-E is proved through A-D, and the ground is more certain than the conclusion.

Hence demonstration by fewer premises is ceteris paribus superior. Now both affirmative and negative demonstration operate through three terms and two premises, but whereas the former assumes only that something is, the latter assumes both that something is and that something else is not, and thus operating through more kinds of premise is inferior.

(2) It has been proved that no conclusion follows if both premises are negative, but that one must be negative, the other affirmative. So we are compelled to lay down the following additional rule: as the demonstration expands, the affirmative premises must increase in number, but there

cannot be more than one negative premise in each complete proof. Thus, suppose no B is A, and all C is B. Then if both the premises are to be again expanded, a middle must be interposed. Let us interpose D between A and B, and E between B and C. Then clearly E is affirmatively related to B and C, while D is affirmatively related to B but negatively to A; for all B is D, but there must be no D which is A. Thus there proves to be a single negative premise, A-D. In the further prosyllogisms too it is the same, because in the terms of an affirmative syllogism the middle is always related affirmatively to both extremes; in a negative syllogism it must be negatively related only to one of them, and so this negation comes to be a single negative premise, the other premises being affirmative. If, then, that through which a truth is proved is a better known and more certain truth, and if the negative proposition is proved through the affirmative and not vice versa, affirmative demonstration, being prior and better known and more certain, will be superior.

(3) The basic truth of demonstrative syllogism is the universal immediate premise, and the universal premise asserts in affirmative demonstration and in negative denies: and the affirmative proposition is prior to and better known than the negative (since affirmation explains denial and is prior to denial, just as being is prior to not-being). It follows that the basic premise of affirmative demonstration is superior to that of negative demonstration, and the demonstration which uses superior basic premises is superior.

(4) Affirmative demonstration is more of the nature of a basic form of proof, because it is a sine qua non of negative demonstration.

Chapter 26

Since affirmative demonstration is superior to negative, it is clearly superior also to reductio ad impossibile. We must first make certain what is the difference between negative demonstration and reductio ad impossibile. Let us suppose that no B is A, and that all C is B: the conclusion necessarily follows that no C is A. If these premises are assumed, therefore, the negative demonstration that no C is A is direct. Reductio ad impossibile, on the other hand, proceeds as follows. Supposing we are to prove that does not inhere in B, we have to assume that it does inhere, and further that B inheres in C, with the resulting inference that A inheres in C. This we have to suppose a known and admitted impossibility; and we then infer that A cannot inhere in B. Thus if the inherence of B in C is not questioned, A's inherence in B is impossible.

The order of the terms is the same in both proofs: they differ according to which of the negative propositions is the better known, the one denying A of B or the one denying A of C. When the falsity of the conclusion is the better known, we use reductio ad impossible; when the major premise of the syllogism is the more obvious, we use direct demonstration. All the same the proposition denying A of B is, in the order of being, prior to that denying A of C; for premises are prior to the conclusion which follows from them, and 'no C is A' is the conclusion, 'no B is A' one of its premises. For the destructive result of reductio ad impossibile is not a proper conclusion, nor are its antecedents proper premises. On the contrary: the constituents of syllogism are premises related to one another as whole to part or part to whole, whereas the premises A-C and A-B are not thus related to one another. Now the superior demonstration is that which proceeds from better known and prior premises, and while both these forms depend for credence on the not-being of something, yet the source of the one is prior to that of the other. Therefore negative demonstration will have an unqualified superiority to reductio ad impossibile, and affirmative demonstration, being superior to negative, will consequently be superior also to reductio ad impossibile.

Chapter 27

The science which is knowledge at once of the fact and of the reasoned fact, not of the fact by itself without the reasoned fact, is the more exact and the prior science.

A science such as arithmetic, which is not a science of properties qua inhering in a substratum, is more exact than and prior to a science like harmonics, which is a science of properties inhering in a substratum; and similarly a science like arithmetic, which is constituted of fewer basic elements, is more exact than and prior to geometry, which requires additional elements. What I mean by 'additional elements' is this: a unit is substance without position, while a point is substance with position; the latter contains an additional element.

Chapter 28

A single science is one whose domain is a single genus, viz. all the subjects constituted out of the primary entities of the genus-i.e. the parts of this total subject-and their essential properties.

One science differs from another when their basic truths have neither a common source nor are derived those of the one science from those the other. This is verified when we reach the indemonstrable premises of a science, for they must be within one genus with its conclusions: and this again is verified if the conclusions proved by means of them fall within one genus-i.e. are homogeneous.

Chapter 29

One can have several demonstrations of the same connection not only by taking from the same series of predication middles which are other than the immediately cohering term e.g. by taking C, D, and F severally to prove A-B--but also by taking a middle from another series. Thus let A be change, D alteration of a property, B feeling pleasure, and G relaxation. We can then without falsehood predicate D of B and A of D, for he who is pleased suffers alteration of a property, and that which alters a property changes. Again, we can predicate A of G without falsehood, and G of B; for to feel pleasure is to relax, and to relax is to change. So the conclusion can be drawn through middles which are different, i.e. not in the same series-yet not so that neither of these middles is predicable of the other, for they must both be attributable to some one subject.

A further point worth investigating is how many ways of proving the same conclusion can be obtained by varying the figure,

Chapter 30

There is no knowledge by demonstration of chance conjunctions; for chance conjunctions exist neither by necessity nor as general connections but comprise what comes to be as something distinct from these. Now demonstration is concerned only with one or other of these two; for all reasoning proceeds from necessary or general premises, the conclusion being necessary if the premises are necessary and general if the premises are general. Consequently, if chance conjunctions are neither general nor necessary, they are not demonstrable.

Chapter 31

Scientific knowledge is not possible through the act of perception. Even if perception as a faculty is of 'the such' and not merely of a 'this somewhat', yet one must at any rate actually perceive a 'this somewhat', and at a definite present place and time: but that which is commensurately universal and true in all cases one cannot perceive, since it is not 'this' and it is not 'now'; if it were, it would not be commensurately universal-the term we apply to what is always and everywhere. Seeing, therefore, that demonstrations are commensurately universal and universals imperceptible, we clearly cannot obtain scientific knowledge by the act of perception: nay, it is obvious that even if it were possible to perceive that a triangle has its angles equal to two right angles, we should still be looking for a demonstration-we should not (as some say) possess knowledge of it; for perception must be of a particular, whereas scientific knowledge involves the recognition of the commensurate universal. So if we were on the moon, and saw the earth shutting out the sun's light, we should not know the cause of the eclipse: we should perceive the present fact of the eclipse, but not the reasoned fact at all, since the act of perception is not of the commensurate universal. I do not, of course, deny that by watching the frequent recurrence of this event we might, after tracking the commensurate universal, possess a demonstration, for the commensurate universal is elicited from the several groups of singulars.

The commensurate universal is precious because it makes clear the cause; so that in the case of facts like these which have a cause other than themselves universal knowledge is more precious than sense-perceptions and than intuition. (As regards primary truths there is of course a different account to be given.) Hence it is clear that knowledge of things demonstrable cannot be acquired by perception, unless the term perception is applied to the possession of scientific knowledge through demonstration. Nevertheless certain points do arise with regard to connections to be proved which are referred for their explanation to a failure in sense-perception: there are cases when an act of vision would terminate our inquiry, not because in seeing we should be knowing, but because we should have elicited the universal from seeing; if, for example, we saw the pores in the glass and the light passing through, the reason of the kindling would be clear to us because we should at the same time see it in each instance and intuit that it must be so in all instances.

Chapter 32

All syllogisms cannot have the same basic truths. This may be shown first of all by the following dialectical considerations. (1) Some syllogisms are true and some false: for though a true inference is possible from false premises, yet this occurs once only-I mean if A for instance, is truly predicable of C, but B, the middle, is false, both A-B and B-C being false; nevertheless, if middles are taken to prove these premises, they will be false because every conclusion which is a falsehood has false premises, while true conclusions have true premises, and false and true differ in kind. Then again, (2) falsehoods are not all derived from a single identical set of principles: there are falsehoods which are the contraries of one another and cannot coexist, e.g. 'justice is injustice', and 'justice is cowardice'; 'man is horse', and 'man is ox'; 'the equal is greater', and 'the equal is less.' From established principles we may argue the case as follows, confining-ourselves therefore to true conclusions. Not even all these are inferred from the same basic truths; many of them in fact have basic truths which differ generically and are not transferable; units, for instance, which are without position, cannot take the place of points, which have position. The transferred terms could only fit in as middle terms or as major or minor terms, or else have some of the other terms between them, others outside them.

Nor can any of the common axioms-such, I mean, as the law of excluded middle-serve as premises for the proof of all conclusions. For the kinds of being are different, and some attributes attach to quanta and some to qualia only; and proof is achieved by means of the common axioms taken in conjunction with these several kinds and their attributes.

Again, it is not true that the basic truths are much fewer than the conclusions, for the basic truths are the premises, and the premises are formed by the apposition of a fresh extreme term or the interposition of a fresh middle. Moreover, the number of conclusions is indefinite, though the number of middle terms is finite; and lastly some of the basic truths are necessary, others variable.

Looking at it in this way we see that, since the number of conclusions is indefinite, the basic truths cannot be identical or limited in number. If, on the other hand, identity is used in another sense, and it is said, e.g. 'these and no other are the fundamental truths of geometry, these the fundamentals of calculation, these again of medicine'; would the statement mean anything except that the sciences have basic truths? To call them identical because they are self-identical is absurd, since everything can be identified with everything in that sense of identity. Nor again can the contention that all conclusions have the same basic truths mean that from the mass of all possible premises any conclusion may be drawn. That would be exceedingly naive, for it is not the case in the clearly evident mathematical sciences, nor is it possible in analysis, since it is the immediate premises which are the basic truths, and a fresh conclusion is only formed by the addition of a new immediate premise: but if it be admitted that it is these primary immediate premises which are basic truths, each subject-genus will provide one basic truth. If, however, it is not argued that from the mass of all possible premises any conclusion may be proved, nor yet admitted that basic truths differ so as to be generically different for each science, it remains to consider the possibility that, while the basic truths of all knowledge are within one genus, special premises are required to prove special conclusions. But that this cannot be the case has been shown by our proof that the basic truths of things generically different themselves differ generically. For fundamental truths are of two kinds, those which are premises of demonstration and the subject-genus; and though the former are common, the latter-number, for instance, and magnitude-are peculiar.

Chapter 33

Scientific knowledge and its object differ from opinion and the object of opinion in that scientific knowledge is commensurately universal and proceeds by necessary connections, and that which is necessary cannot be otherwise. So though there are things which are true and real and yet can be otherwise, scientific knowledge clearly does not concern them: if it did, things which can be otherwise would be incapable of being otherwise. Nor are they any concern of rational intuition-by rational intuition I mean an originative source of scientific knowledge-nor of indemonstrable knowledge, which is the grasping of the immediate premise. Since then rational intuition, science, and opinion, and what is revealed by these terms, are the only things that can be 'true', it follows that it is opinion that is concerned with that which may be true or false, and can be otherwise: opinion in fact is the grasp of a premise which is immediate but not necessary. This view also fits the observed facts, for opinion is unstable, and so is the kind of being we have described as its object. Besides, when a man thinks a truth incapable of being otherwise he always thinks that he knows it, never that he opines it. He thinks that he opines when he thinks that a connection, though actually so, may quite easily be otherwise; for he believes that such is the proper object of opinion, while the necessary is the object of knowledge.

In what sense, then, can the same thing be the object of both opinion and knowledge? And if any one chooses to maintain that all that he knows he can also opine, why should not opinion be knowledge? For he that knows and he that opines will follow the same train of thought through the same middle terms until the immediate premises are reached; because it is possible to opine not only the fact but also the reasoned fact, and the reason is the middle term; so that, since the former knows, he that opines also has knowledge.

The truth perhaps is that if a man grasp truths that cannot be other than they are, in the way in which he grasps the definitions through which demonstrations take place, he will have not opinion but knowledge: if on the other hand he apprehends these attributes as inhering in their subjects, but not in virtue of the subjects' substance and essential nature possesses opinion and not genuine knowledge; and his opinion, if obtained through immediate premises, will be both of the fact and of the reasoned fact; if not so obtained, of the fact alone. The object of opinion and knowledge is not quite identical; it is only in a sense identical, just as the object of true and false opinion is in a sense identical. The sense in which some maintain that true and false opinion can have the same object leads them to embrace many strange doctrines, particularly the doctrine that what a man opines falsely he does not opine at all. There are really many senses of 'identical', and in one sense the object of true and false opinion can be the same, in another it cannot. Thus, to have a true opinion that the diagonal is commensurate with the side would be absurd: but because the diagonal with which they are both concerned is the same, the two opinions have objects so far the same: on the other hand, as regards their essential definable nature these objects differ. The identity of the objects of knowledge and opinion is similar. Knowledge is the apprehension of, e.g. the attribute 'animal' as incapable of being otherwise, opinion the apprehension of 'animal' as capable of being otherwise-e.g. the apprehension that animal is an element in the essential nature of man is knowledge; the apprehension of animal as predicable of man but not as an element in man's essential nature is opinion: man is the subject in both judgements, but the mode of inherence differs.

This also shows that one cannot opine and know the same thing simultaneously; for then one would apprehend the same thing as both capable and incapable of being otherwise-an impossibility. Knowledge and opinion of the same thing can co-exist in two different people in the sense we have explained, but not simultaneously in the same person. That would involve a man's simultaneously apprehending, e.g. (1) that man is essentially animal-i.e. cannot be other than animal-and (2) that man is not essentially animal, that is, we may assume, may be other than animal.

Further consideration of modes of thinking and their distribution under the heads of discursive thought, intuition, science, art, practical wisdom, and metaphysical thinking, belongs rather partly to natural science, partly to moral philosophy.

Chapter 34

Quick wit is a faculty of hitting upon the middle term instantaneously. It would be exemplified by a man who saw that the moon has her bright side always turned towards the sun, and quickly grasped the cause of this, namely that she borrows her light from him; or observed somebody in conversation with a man of wealth and divined that he was borrowing money, or that the friendship of these people sprang from a common enmity. In all these instances he has seen the major and minor terms and then grasped the causes, the middle terms.

Let A represent 'bright side turned sunward', B 'lighted from the sun', C the moon. Then B, 'lighted from the sun' is predicable of C, the moon, and A, 'having her bright side towards the source of her light', is predicable of B. So A is predicable of C through B.